

Interesting and correct results

navstokgap, September 2026: the results
whose derivations the scorer could follow to
the end, condensed

Alejandro Rivero (director); notes written with
Claude and Codex agents; scored by Claude Opus
5.5, reviewed by Claude Fable 5.1

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About this digest

The repository navstokgap (source at github.com/arivero/navstokgap, every note and PDF at arivero.github.io/navstokgap) is a research programme opened on 6 September 2026 and directed by Alejandro Rivero. Its derivations were written by language-model agents, Claude and GPT-6 Astra working through Codex, under written rules, with adversarial review rounds by further model instances recorded in the notes. The programme asks what survives refinement in two problems.

Newton's action problem. In Book I of the *Principia*, Newton compares inertial motion with motion under a constant force and takes their difference, the sagitta or the area between the inertial line and Galileo's parabola, to zero (Lemmas X and XI, Proposition I). The

programme asks whether that limit can still be taken once the comparison has to be recorded by a physical instrument, and whether a positive floor on the action spent, of the order of Planck's constant, is then forced. The proved results are theorems about marking and recording models with explicit premises. Several of them need a positive quantum of action as an input; the independent argument that Newton-age premises force $\hbar > 0$ remains open, and the notes say so.

The mass gap of pure $SU(3)$ Yang–Mills theory. In four dimensions and in the Jaffe–Witten formulation, the programme works in theorem-sized steps on the lattice: finite-volume gaps, volume-uniform strong-coupling gaps with explicit constants, exact identities for a single lattice halving, and the small-field estimates a continuum argument would need. The continuum clause $m > 0$ remains open. No entry in this digest claims a solution of the Millennium problem.

A third thread compares these two limits with QCD at zero and nonzero quark mass; it is the frame of the eventual paper and the source of several mechanisms, and it has no construction programme of its own.

How the selection was made

On 1 October 2026 the repository held 175 research notes, about 330,000 words. Each note was read in full by Claude Opus 5.5 and scored on two scales from 1 to 10 under one fixed rubric. *Interestingness* asks whether an expert outside the project would want to know the result: 9 or 10 for a theorem with a surprising conclusion or a precise closure of a natural route, 7 or 8 for a solid nontrivial theorem, construction or decisive obstruction with explicit constants, 5 or 6 for useful supporting work, below that for bookkeeping. *Correctness* asks whether the note's own derivations support what its current text claims: 9 or 10 for complete proofs at the stated level with every hypothesis stated where it is used, 7 or 8 for essentially correct with minor gaps, 5 or 6 where a step is unjustified or claims outrun proofs, below that for a substantive error or an unmarked superseded claim. The scorer also wrote the condensed statement printed under each heading.

Claude Fable 5.1 then reviewed every selected statement against the note it condenses and against the repository's list of errors already caught and corrected. Entries whose main claim was found false, or superseded without acknowledgment, were removed from both volumes and are marked in the appendix. Statements that overstated a result were cor-

rected, and a reviewer's note was added where the reader should know a limit the statement does not make visible.

The scores are a model's reading, with the scorer's reasons kept on file; they are not peer review. A high correctness score means that the derivation is complete at the level the note states, which may be conditional, formal or a reading of sources. It confers no literature novelty and settles no physical question.

This volume keeps the notes scored 6 or more in interestingness and 8 or more in correctness after that review, 22 notes in all. Every entry here is a derivation the scorer could follow to the end at the level the note states, and the reviewer found no false claim in its condensed statement. Conditional and formal results appear with their status named; a conditional theorem is complete as a theorem, and its hypothesis is open. The companion volume, *The most interesting results*, collects all 26 notes scored 6 or more in interestingness, including those whose derivations are incomplete.

How to read an entry

Each entry opens with a one-line headline, then the note's original title and file name, the two scores, the note's proof status (proved, conditional, conjectural, mixed) and its kind (the-

orem, construction, obstruction, calculation, reading, map). The condensed statement follows, then the address of the full note on the project site, where the Markdown source and a typeset PDF are linked. Within each part the entries are ordered by interestingness, then by correctness.

Conventions

Constants are explicit throughout: $\hbar$, c and k_B are never set to one. For a constant force F acting for a time τ on a mass m , the sagitta is $s = F\tau^2/(2m)$ and the impulse is $J = F\tau$. An *action floor* is a positive lower bound on an action-valued observable over all admitted preparations; the *Planck gap* is the project's name for such a floor in Newton's comparison. A *mark* or *record* is the registration of a comparison by an instrument, modelled with explicit error and disturbance; κ denotes the product of a mark's imprecision and its kick, and the Gaussian floor ζ is the least value of $\sqrt{\det \Sigma}$ for an admitted covariance Σ .

On the gauge side, D is the spacetime dimension, a the lattice spacing and g the coupling. The heat-kernel lattice theory has heat time $t = \lambda_D a^{4-D}$ per plaquette with $\lambda_D = \hbar g_{\text{cl}}^2$, so in $D = 4$ the parameter $t = g^2$ contains $\hbar$. A *halving* is the refinement of a lattice by inserting a new layer of edges; a *cut at fraction* s inserts it off centre. The *halving atlas* is

the project's table of what one halving does in each dimension and for each group. The *target box* is the strong-coupling region in which a volume-uniform gap is proved. The *mid-plane* is the $(D - 1)$ -dimensional gauge theory that carries the renormalization of one directional halving.

Newton's action problem and the Planck gap

Conjugate mark error and recoil make invariant Gaussian tests need about $65\bar{h}$

The mark cost is exactly $\bar{h}/2$, and the worst-case theorem survives as a statistical one

Note mark-cost-and-statistical-floor. Interest 7/10, correctness 8/10; proved theorem.

A position mark realized by the impulsive von Neumann coupling $\lambda\hat{y} \otimes \hat{P}_A$ has error operator $\hat{N} = \hat{Q}_A/\lambda$ and delivered impulse $\hat{D} = -\lambda\hat{P}_A$ with $[\hat{N}, \hat{D}] = -i\bar{h}$, so the mark cost $\kappa = \delta\Delta$ (standard deviations of error and recoil) is at least $\bar{h}/2$ for every probe state, independent of λ . In a Gaussian model of non-adaptive marks at arbitrary times, with the body's initial position and velocity unknown and tests required to be invariant under them, the optimal test's deflection obeys $d^2 \leq F^2\tau^3/(24m\kappa)$, sharp in the dense limit. Deciding between inertial motion and constant force F over time τ at error

probability ϵ then needs $\tau \Delta E = F^2 \tau^3 / (2m) \geq 48 z_{1-\epsilon}^2 \kappa$, that is $24 z^2 \hbar$ (about $65 \hbar$ at 5 per cent) for uncorrelated error and recoil. The proof bounds the noise by an arithmetic-geometric mean step, reduces it to a Dirichlet energy, and uses the Poincare constant $\tau^3/12$ for a function vanishing at both ends. For Gaussian quantum probes the Wigner function is a density, and a correlation ρ between error and recoil multiplies the floor by $\sqrt{(1-\rho)/(1+\rho)}$; three marks at $0, \tau/2, \tau$ reach this within $4/3$, so as $\rho \rightarrow 1$ the floor disappears (Yuen's contractive-state mechanism applied to the meter). A protocol that knows the body's preparation escapes the floor, and non-Gaussian grid probes escape it too. The note identifies Newton's premise M3 (irreducible fits-interval times corpuscle impulse) with Robertson's inequality for the corpuscle as a labelled reading. It claims no bound for arbitrary instruments or adaptive protocols.

Full note: <https://arivero.github.io/navs tokgap/notes/mark-cost-and-statistical-floor.html>

Adaptive quantum protocols obey $JL + sP \geq \hbar \arcsin(1 - 2\epsilon)$; unbounded apertures evade it

The probabilistic Planck gap: $F\tau(L + \tau P/2m) \geq \hbar \arcsin(1 - 2\epsilon)$, and why it is resource-relative

Note planck-gap-probabilistic. Interest

7/10, correctness 8/10; proved theorem.

Under the inertial and constant-force hypotheses a quantum particle's evolutions differ by a phase-space displacement whose position component is the sagitta $F\tau^2/2m$ and whose momentum component is the impulse $F\tau$; their product is $\tau\Delta E$. If every state the body passes through has position spread at most L and momentum spread at most P , then any measurement deciding the hypotheses with error at most ϵ needs $F\tau L/\hbar + F\tau^2 P/(2m\hbar) \geq \arcsin(1 - 2\epsilon)$. This holds single-shot by Helstrom and Mandelstam-Tamm, and for every finite adaptive protocol of instruments with any Kraus rank and memory, by a hybrid argument; the constant improves from $1 - 2\epsilon$ to the arcsine via a companion note. Universality holds because inserting instruments subdivides a monotone displacement path without increasing its total variation. At aperture area $A = LP$ and aspect r the floor is

$$\tau\Delta E \geq \arcsin^2(1 - 2\epsilon)\hbar^2/(A(r + r^{-1})^2)$$

, so a fixed area alone gives no floor: squeezing drives it to zero. Two narrow packets separated by $\pi\hbar/(F\tau)$ decide with error $O(\tau\Delta E/\hbar)$ at arbitrarily small $\tau\Delta E$, saturating the theorem. The floor therefore prices the size of the laboratory, and universality over preparations fails. The inequality's shape is the standard quantum limit; the note uses no error-disturbance relation.

Full note: <https://arivero.github.io/navs/tokgap/notes/planck-gap-probabilistic.html>

Classical shared-noise recording breaks the phase-area floor; Gaussian closure holds only for affine copies

Shared radiation and closure under recording

Note sed-closure-under-recording. Interest 7/10, correctness 8/10; mixed obstruction.

The note asks whether a classical common radiation background with zero-point spectrum $\kappa\omega$ per mode, as in stochastic electrodynamics, can enforce a floor $\sqrt{\det \Sigma} \geq \kappa$ on the posterior phase-space area of a recorded body. Theorem 1 shows that it cannot when two pointers with retained classical coordinate readouts are allowed: for any finite joint covariance the posterior area is at most $\sigma_Y \sigma_P / G + \sigma_Y \sigma_{Z_2} / B$, so it tends to zero with the gains, and Theorem 2 keeps this with radiation damping at fixed cutoff for short pulses. A Gaussian restriction $V + i\kappa\Omega \succeq 0$ on every retained apparatus restores closure (Proposition 3); at $\kappa = \hbar/2$ this is Gaussian quantum measurement theory. Later sections give exact Gaussian results: a side monitor of the kick breaks a saturated readout law unless its conditional determinant stays at least κ^2 ; Hamiltonian copies into physical memories give posterior closure, partition-independent terminal chan-

nels and a continuous body–record limit with retained recoil. Nonlinear copies then fail: a quadratic or bounded C^2 copy produces posteriors below the floor on positive-probability records under every subdivision, while the quantum reference compensates through a curvature term whose generator excludes positive classical transition kernels. Two-packet examples show local jets and Fisher data omit predictive coherence. A shared-score-sign classical preparation reproduces the correction but fails full pointer readout and Newtonian free transport, and a Zig-Zag switching repair needs preparation-dependent forces. Nothing here selects $\kappa > 0$; the zero branch stays admissible.

Full note: <https://arivero.github.io/navs tokgap/notes/sed-closure-under-recording.html>

Galileo cell action $F^2\tau^3/(24m)$ is a Schauder-additive measure on cuts; marks spend exactly it

Newton's cell action as a measure on cuts

Note cut-measure-newton. Interest 6/10, correctness 9/10; proved theorem.

A constant force F acting on a mass m over a time cell τ separates the parabola from the inertial chord by $\delta x = Ft(\tau - t)/(2m)$. Its kinetic action, the Galileo cell action, is

$K_\tau = \frac{m}{2} \int \dot{\delta x}^2 = F^2 \tau^3 / (24m)$. A cut at fraction s spends exactly $3s(1 - s)K_\tau$, the energy of the inserted Schauder hat, and successive hats are energy-orthogonal. Along any nested sequence of cuts the spent total is $K_\tau - \sum_i K_{\tau_i}$, which tends to K_τ exactly when the mesh tends to zero: Newton's refinement is the Lévy–Ciesielski expansion. Halving only the remaining piece, the Zhuangzi stick schedule, spends $\frac{6}{7}K_\tau$; two quarter-cuts a day spend $\frac{27}{28}K_\tau$. Under the free-particle Wiener measure with variance rate $\hbar/m$, each cut's Kullback–Leibler divergence is its share divided by $\hbar$. The optimal equal-prior test of force against no force errs with probability $\Phi(-\sqrt{K_\tau/2\hbar})$, reproducing a mark-mesh threshold $K_\tau \geq 2z_{1-\epsilon}^2 \hbar$. Proposition 7 shows that the best discernibility of position marks with $\delta\Delta \geq \kappa$ at given cut times equals the spent share over κ . All results assume $\hbar > 0$ or a floor κ ; the note does not derive the floor.

Full note: <https://arivero.github.io/navs tokgap/notes/cut-measure-newton.html>

Newton's chord versus Galileo's parabola carries action $F^2 \tau^3 / (24m)$; its coherent record vanishes under refinement

Galileo's comparison as two-path interference

Note galileo-two-path-interference. Interest 6/10, correctness 9/10; conditional theorem.

For transverse motion under constant force F over a cell of duration τ , the impulsive chord (Newton's polygon step) and the constant-force parabola (Galileo's fall), matched at both endpoints, differ in action by $K_\tau = F^2\tau^3/(24m) = (F/4v)A = \tau\Delta E/12$, where $A = vF\tau^3/(6m)$ is the area between the inertial tangent and the parabola and ΔE the kinetic energy gained. With canonical quantum kinematics at action scale ε the two propagators satisfy $Q_\tau = e^{iK_\tau/\varepsilon}U_\tau$ exactly. In a coherently controlled two-arm comparison with n independent passes, the optimal equal-prior error is $\frac{1}{2}(1 - \sqrt{1 - \cos^{2n}(K_\tau/2\varepsilon)})$, so error at most η requires $K_\tau \geq \varepsilon d_{n,\eta}$ on the first lobe, with $d_{1,\eta} = 2 \arcsin(1 - 2\eta)$. At fixed ε , refinement $\tau \rightarrow 0$ sends the error to $1/2$; at fixed τ , $\varepsilon \rightarrow 0$ has no limit. A relative average over inverse resolution, or a tunable attenuated comparison, gives explicit non-commuting iterated limits. All thresholds scale with the supplied ε ; the note does not derive a positive scale and states that the Born rule, coherence and calibration are premises.

Full note: <https://arivero.github.io/navs tokgap/notes/galileo-two-path-interference.html>

Newton's inscribed polygon and the parabola differ quantum mechanically by the phase $F^2 \sum_j \tau_j^3 / (24m\hbar)$

Newton's inscribed polygon differs from the parabola by the phase of its segments

Note polygon-lift-phase. Interest 6/10, correctness 9/10; proved theorem.

For a body of mass m under constant transverse force F , partition $[0, \tau]$ into steps τ_j and let Newton's polygon move uniformly along each chord of the parabola, with each step's impulse split equally between its ends. Classically the polygon and the parabola coincide in state at every vertex. Using only the Weyl relations, their quantum evolutions differ by a pure phase, $W_{\text{poly}} = e^{i\theta_N} W_F$ with $\theta_N = F^2 \sum_j \tau_j^3 / (24m\hbar) = (F/2v\hbar) \sum_j S_j$, where S_j is the parabolic segment under the j -th chord at horizontal speed v . Inserting a vertex at fraction λ lowers the phase by $F^2 \tau_j^3 \lambda(1 - \lambda) / (8m\hbar)$, the inscribed triangle over $\hbar$, so halving removes it in the proportions of Archimedes' quadrature. Reversing the order of the step operators gives a holonomy $\Phi_N = F^2(\tau^3 - \sum \tau_j^3) / (6m\hbar)$, and $\Phi_N + 4\theta_N = \tau \Delta E / 3\hbar$ for every partition. The phase is a c-number, so no squeezing or mixing of the body changes it; a control qubit resolves one step at error ϵ only if $\tau_j \geq (48m\hbar \arcsin(1 - 2\epsilon) / F^2)^{1/3}$. For any force history the phase is $\sum_j \mathcal{K}_{\tau_j}[f] / \hbar$, with $\mathcal{K}$ the kinetic action relative to the chord, and un-

correlated Gaussian marks obey $d^2 \leq \mathcal{K}_\tau/\kappa$, so at $\kappa = \bar{h}/2$ the best deflection is twice the chord phase. A Newton-age analogue counts fits along the path; the continuous-speed extension is the note's.

Full note: <https://arivero.github.io/navs tokgap/notes/polygon-lift-phase.html>

In the mark model, best verdicts are continuous at zero force exactly when $\kappa > 0$

Leibniz's law of continuity, read on records, selects a positive action floor for motions

Note leibniz-continuity-records. Interest 6/10, correctness 8/10; conditional reading.

In a Gaussian mark model where each position mark has error δ_j and imparts an independent impulse of spread Δ_j with $\delta_j \Delta_j \geq \kappa$, consider deciding inertia against a constant force F over a cell of duration τ for a body of mass m . Protocols are non-adaptive and tests are invariant under unknown initial position and velocity. The best achievable error is $P_*(F) = \Phi(-\frac{1}{2}\sqrt{K_\tau/\kappa})$ with $K_\tau = F^2\tau^3/(24m)$ when $\kappa > 0$, and 0 for every $F \neq 0$ when $\kappa = 0$. So the best verdict is continuous in F at $F = 0$ exactly when $\kappa > 0$; the same holds for rest against uniform motion, while static shapes are decided arbitrarily well for every κ . With bounded apertures, an observer who knows the preparation reaches the

same dichotomy with $\hbar$. The note reads this against Leibniz's 1687 law of continuity, measuring outcomes by discernibility, a transfer it labels as its own, and supplies quoted readings of Leibniz, Newton and Galileo with their commitments. It does not derive $\kappa > 0$ from Newton-age premises; it locates the fork.

Full note: <https://arivero.github.io/navs/tokgap/notes/leibniz-continuity-records.html>

A classical statistical-speed premise reproduces the quantum record floor; Gaussian restriction gives $h_* = 2\zeta$

Newton indeterminacy routes: the physical premise a record floor needs

Note newton-indeterminacy-routes. Interest 6/10, correctness 8/10; conditional theorem.

The note asks which physical premise turns Newton's comparison of an inertial line with a constant-force parabola into a positive action floor on records. Theorem F: if every mark and recording apparatus obeys a statistical-speed bound $\frac{1}{2} \|\{A, \log \rho\}\|_{L^2(\rho)} \leq \Delta_\rho A / h_*$ for the Hamiltonian generators used, on a state class closed under the needed compositions, then deciding between the hypotheses at error

ϵ needs

$$\begin{aligned} \frac{s}{8} \sum_j \sup \Delta D_j + \frac{J}{2} \sum_j \sup \Delta X_j \\ \geq h_* \arcsin(1 - 2\epsilon) \end{aligned}$$

, with sagitta $s = F\tau^2/2m$ and impulse $J = F\tau$; this matches the quantum disturbance theorem. Lemma G: Gaussian ensembles whose symplectic eigenvalues are at least ζ satisfy the premise for affine generators with $h_* = 2\zeta$, sharp. A $\sin(kq)$ generator shows that no Gaussian covariance restriction controls all nonlinear instruments. A stochastic-electrodynamics background with mode energy $\zeta\omega$ supplies a stationary covariance but not closure under recording, since body observations sharpen posteriors without bound. Newton's inflexion observations, read at passage level in the 1718 and 1730 editions, show apparatus-dependent bending, and even a law $\theta b = C\Lambda$ is a deterministic canonical kick. The note does not justify the premise or fix ζ in terms of a radiation unit.

Full note: <https://arivero.github.io/navs tokgap/notes/newton-indeterminacy-routes.html>

Joint determinacy of place and motion is the Principia statement whose denial introduces $\bar{h}$

A fifth postulate for the Principia: joint determinacy of place and motion

Note principia-fifth-postulate. Interest 6/10, correctness 8/10; proved theorem.

The note proposes joint determinacy, the existence of states in which a body's place and quantity of motion are both sharp at one instant, as the *Principia* statement whose denial gives mechanics with $\bar{h} > 0$, anchored in the Book I Section I scholium on the ultimate velocity. Theorem A: for every real $\bar{h}$ the Moyal product satisfies Newton's second law as an operator identity, and for quadratic potentials (inertia, uniform gravity, Proposition X) every observable evolves classically, so commutativity is independent of these Heisenberg-form axioms. Theorem B (Gutt, Fletcher): an associative unital product on polynomial observables, compatible with conjugation and covariant under the affine symplectic group, is Moyal with one real constant $\bar{h}$; a deformed product shows the covariance premise is extra to the *Principia*. Theorem B': an affine-symplectic-invariant, noise-closed set of Gaussian states is $\sqrt{\det \Sigma} \geq \zeta$, a second one-constant denial within a commutative theory. Theorem C: for $\bar{h} \neq 0$ the recorded Galileo comparison obeys the disturbance floor; at $\bar{h} = 0$ with all states

admissible there is none. Theorem D: dilations are automorphisms only at $\hbar = 0$, mirroring Wallis's similar-figures postulate. Theorem E (Landau-Pollak-Slepian): confining position and momentum to half-widths a, b with probability $1 - \epsilon$ needs $\lambda_0(ab/|\hbar|) \geq (1 - 2\epsilon)^2$. The analogy is partial: the floor also arises from a state restriction without noncommutativity.

Full note: <https://arivero.github.io/navstokgap/notes/principia-fifth-postulate.html>

Any unitary marking protocol testing force against free motion pays an $\hbar$ -scale disturbance floor

The record costs disturbance: a floor for every instrument

Note record-costs-disturbance. Interest 6/10, correctness 8/10; proved theorem.

A body of mass m moves on $[0, \tau]$ either freely or under a constant force F ; write $s = F\tau^2/(2m)$ for the sagitta and $J = F\tau$ for the impulse. An experimenter applies impulsive marks, each an arbitrary unitary U_j on body and apparatus, possibly adaptive, followed by pointer readings, and must decide between the two hypotheses with equal-prior error at most ϵ for every pair of initial states. Define each mark's disturbances $\hat{D}_j = U_j^\dagger \hat{p} U_j - \hat{p}$ and $\hat{X}_j = U_j^\dagger \hat{y} U_j - \hat{y}$.

Theorem U states that for every comparison line $a + bt$, $1 - 2\epsilon$ is bounded by $\bar{h}^{-1} \sum_j \sup \Delta(c_j \hat{D}_j - mc'_j \hat{X}_j)$, where $c(t)$ is the force displacement minus the line and the suprema run over the states entering mark j in a chain of hybrid processes. Choosing the Chebyshev line gives

$$\frac{s}{8} \sum_j \Delta(\hat{D}_j) + \frac{J}{2} \sum_j \Delta(\hat{X}_j) \geq \bar{h} (1 - 2\epsilon),$$

improved to $\bar{h} \arcsin(1 - 2\epsilon)$ by a Bures-angle version proved elsewhere. The proof translates the offset between hypotheses through each mark and bounds each step by the Mandelstam–Tamm inequality; the reading error never enters. Von Neumann position marks reduce it to a pure recoil bound, and momentum-reading marks pay in position jumps paired with the impulse. The note claims the combination and the Newtonian reading, with standard ingredients credited to Ozawa, Marvian–Spekkens and Tajima et al. It does not prove attainment, it treats finite-duration couplings only by a slicing remark, and $\bar{h}$ enters as supplied canonical kinematics.

Full note: <https://arivero.github.io/navstokgap/notes/record-costs-disturbance.html>

Momentum-transfer marks with arbitrary probe states need sagitta times total recoil at least $8\hbar$

The record costs recoil: a floor for every probe state

Note record-costs-recoil. Interest 6/10, correctness 8/10; proved theorem.

A body of mass m is tested over $[0, \tau]$ for free motion against a constant force F , by marks that couple position to probe pointers through $e^{-i\lambda_j \hat{y} \hat{P}_j / \hbar}$; the probes may be in any states, Gaussian, squeezed, correlated, mixed or grid. Let $\Delta_j = \lambda_j \Delta \hat{P}_j$ be the spread of the impulse mark j delivers to the body and $s = F\tau^2/(2m)$ the sagitta. Theorem R states that a test with equal-prior error at most ϵ for every initial body state obeys

$$1 - 2\epsilon \leq \hbar^{-1} \min_{a,b} \sum_j \Delta_j |P(t_j) - a - bt_j|$$

, where P is the forced displacement; for constant force this gives $s \sum_j \Delta_j \geq 8\hbar(1 - 2\epsilon)$, improved elsewhere to $8\hbar \arcsin(1 - 2\epsilon)$, so $2h$ at certain decision. The proof moves the signal into probe translations, which a probe can register only through its momentum spread, and that momentum times λ_j is the recoil. Proposition G shows that approximate Gottesman–Kitaev–Preskill grid probes with zero position–momentum correlation decide the three-mark

comparison at almost every force with vanishing error while recoil diverges, so an earlier floor on second moments holds only for Gaussian probes. A three-mark Gaussian protocol comes within the factor $z_{1-\epsilon}/\arcsin(1-2\epsilon)$, about 1.47 at five per cent error, of Theorem R. The note does not establish attainment as $\epsilon \rightarrow 0$ and covers only momentum-transfer couplings; $\hbar$ is supplied by the quantum kinematics.

Full note: <https://arivero.github.io/navs tokgap/notes/record-costs-recoil.html>

Bures-angle hybrid chains give sharp arcsin constants and a split body/mark force accounting

The record's distance is a path length

Note record-distance-path-length. Interest 6/10, correctness 8/10; proved theorem.

The note sharpens and joins three necessity bounds for deciding, at equal-prior error ϵ , whether a quantum body of mass m felt a force history $f(t)$ on $[0, \tau]$ or moved freely. Theorem P replaces total variation by the Bures angle $A = \arccos F$ in every hybrid-chain proof, using its metric property, monotonicity, Uhlmann purifications and the Fuchs–van de Graaf bound, so each protocol bound holds with $\arcsin(1-2\epsilon)$ in place of $1-2\epsilon$; the recoil bound becomes $s \sum_j \Delta_j \geq 8\hbar \arcsin(1-2\epsilon)$,

which is $2h$ at certain decision, with $s = F\tau^2/(2m)$. Theorem K states that any adaptive instrument protocol with the same preparation under both hypotheses obeys $\hbar \arcsin(1 - 2\epsilon) \leq \int_0^\tau |f(t)|L(t) dt$, with $L(t)$ the supremum of the body's position spread over the chain's conditional states; this implies $F\tau L + sP \geq \hbar \arcsin(1 - 2\epsilon)$ and is attained by a two-packet superposition as $\epsilon \rightarrow 0$. Theorem M splits $f = g + h$ and pays h at the body through $\int |h|L$ and g at the marks through their momentum and position disturbances weighted by an offset c with $c'' = g/m$. For position (von Neumann) marks and a force of one sign this gives $\hbar \arcsin(1 - 2\epsilon) \leq \int |f| \min(L, R)$, with $R(u) = m^{-1} \sum_j \Delta_j G_\tau(t_j, u)$ a recoil length built from the Dirichlet Green's function. The note does not show that the recoil, disturbance or split bounds are attained; the best known recoil protocol sits about 1.47 above at five per cent error, and $\hbar$ enters as supplied quantum kinematics.

Full note: <https://arivero.github.io/navs tokgap/notes/record-distance-path-length.html>

Refinement and the comparison of limits

Exact telegraph return bridge: Bessel count law, Beta midpoint law, closed-form midpoint crossover

A finite-speed return bridge: cuts and midpoint crossover

Note telegraph-return-bridge. Interest 6/10, correctness 9/10; proved calculation.

A particle of mass m moves at speed $u < c$ and reverses direction at the events of a Poisson clock of rate λ . Conditioned to return to its start with reversed velocity after time T , the number of reversals is $2K + 1$ with $\Pr(K = k) = (z/2)^{2k} / [(k!)^2 I_0(z)]$, $z = \lambda T$. Given K , the positive and negative run lengths are independent uniform simplices of total $T/2$. This one probability space evaluates every finite set of cuts, so refinement is exactly consistent. The midpoint Y has an atom $\Pr(Y = uT/2) = 1/I_0(z)$. For $k \geq 1$, $(Q + 1)/2$ with $Q = 2Y/uT$ is Beta($k + 1, k$). Hence $\mathbb{E}Q = \mathbb{E}Q^2 = R(z) = \int_0^z I_0/(zI_0(z))$. The midpoint coefficient $4m \text{Var } Y/T$ equals $H_* g(z)$ with $H_* = mu^2/\lambda$ and $g = zR(1 - R)$. It starts as $z^3/6$, is at most $z/4$, and tends to one as $T \rightarrow \infty$. The unscaled midpoint mean tends to $u/2\lambda$, which corrects an earlier claim that it vanishes. The sampled-polygon kinetic action converges to $mu^2 T/2$ with error at most $(mu^2/2)N_T|\pi|$. A

positive plateau K_* requires $T \geq K_*/mu^2$. A common finite window for arbitrarily small masses therefore forces $K_* = 0$. Setting $H_* = \hbar$, $u = c$ gives Compton scales only as a formal identification. The note claims no quantum dynamics.

Full note: <https://arivero.github.io/navs tokgap/notes/telegraph-return-bridge.html>

2D plaquette-action continuum limits are central Levy semigroups; 4D large-field threshold $2b_0c > 4$

The zero-spacing limit for any plaquette action and any dimension

Note zero-spacing-any-action. Interest 6/10, correctness 8/10; mixed theorem.

A series refinement move convolves plaquette weights and multiplies their character coefficients, so heat kernels are closed under it. A parallel move multiplies weights pointwise, so fixed-potential exponential families such as Wilson's are closed under it. For $U(1)$ the parallel product of heat kernels is a heat kernel at time $st/(s+t)$ plus shifted images of mixture weight $e^{-2\pi^2 m^2/(s+t)}$. In two dimensions, for a compact connected simple group and symmetric class-function weights with finite exponents $\psi(R) = \lim a^{-2}(1 - \hat{c}_R(a))$, every possible continuum limit is a central Levy semigroup

with

$$\psi(R) = \frac{\sigma^2}{2} C_2(R) + \int (1 - \operatorname{Re} \chi_R / d_R) d\nu$$

. Every such semigroup is realized. Yang–Mills is the Gaussian case, selected by a Lindeberg condition. A centre-element atom adds an N -ality-dependent string tension. In D dimensions one refinement step leaves perturbative, large-field and jump errors. Below four dimensions these are summable at fixed coupling. In four dimensions the large-field density per physical volume scales as $(a\Lambda)^{2b_0c-4}$, giving the known strict threshold $c > 96\pi^2/11N$, which is $6/11$ of the instanton action for $SU(2)$. Above four dimensions there is no small-field regime. The note proves no common continuum limit or gap.

Full note: <https://arivero.github.io/navs tokgap/notes/zero-spacing-any-action.html>

Yang–Mills theory and the mass gap

One lattice halving factors into closing series moves and a mid-plane gauge integral; U(1) controlled

Series and parallel: where the gauge insertion law stops closing

Note series-parallel-gauge-refinement. Interest 7/10, correctness 8/10; mixed theorem.

Consider a heat-kernel lattice gauge theory in D dimensions with compact group G , plaquette heat times

$$t_{\mu\nu} = \lambda_D a_\mu a_\nu / \prod_{\rho \neq \mu, \nu} a_\rho$$

and $\lambda_D = \bar{h} g_{\text{cl}}^2$. Halving one lattice direction is shown to factor exactly (Proposition 1): faces containing that direction are cut in two and integrate out exactly by heat-kernel convolution, while each of the $\binom{D-1}{2}$ transverse planes acquires a mid-plane whose integral Ψ is a $(D - 1)$ -dimensional gauge theory with Brownian-bridge edge laws. This explains why two-dimensional Yang–Mills closes and locates the renormalization problem in $D = 3, 4$. For the Gaussian free field the defect against the coarse action is an exact nonpositive quadratic form, bounded relatively by $a^2 K^2/16 + (Ga)^2/8$, and equals the error of Migdal’s bond moving; an off-centre cut scales it by $4s(1 - s)$. For compact $U(1)$ the mid-plane integral is exact, a Villain-type vortex sum, and in three dimensions one unperturbed halving equals the free step up to an extensive error of density $e^{-\pi^2/(8\lambda_3 a)}$ on the small-field set (Theorem 5), for every cut fraction. For non-abelian groups the bridge-midpoint curvature softening is computed, $t|X|^2/512$ per cut face for $SU(3)$, and a finite-cube power bound holds conditional on a uniform Laplace expansion. The note does not prove iteration

stability, a continuum limit, monopole densities or any mass gap, and states the one-step $U(1)$ bound as such.

Full note: <https://arivero.github.io/navs/tokgap/notes/series-parallel-gauge-refinement.html>

Villain $U(1)$: monopole-free measure within TV $O((L/a)^3 e^{-\pi^2/6\lambda_3 a})$ in 3D; fails in 4D

Compact $U(1)$ under refinement: an explicit total-variation bound in three dimensions, and why it fails in four

Note villain-monopole-refinement. Interest 6/10, correctness 9/10; proved theorem.

Consider the heat-kernel (Villain) $U(1)$ lattice gauge theory on a periodic D -dimensional lattice of spacing a and side $L = Na$, with plaquette heat time t . Poisson summation writes the measure exactly as a mixture of classes labelled by monopole charges q and integer torus fluxes m , weighted by

$$\exp\{-(2\pi^2/t)(q^\top (dd^\top)^+ q + |n_{\mathcal{H}}|^2)\}$$

. In three dimensions with $t = \lambda_3 a \leq 1$, the total-variation distance to the monopole-free measure is at most

$$2(L/a)^3 e^{-\pi^2/(6\lambda_3 a)} / (1 - e^{-\pi^2/(2\lambda_3 a)})$$

. This vanishes faster than any power of a at fixed λ_3 and L . A single-pair lower bound

brackets the monopole density. The monopole-free part is the free photon with flux sectors weighted by $e^{-2\pi^2|m|^2/(\lambda_3 L)}$. Pushforward under blocking preserves the bound, so iteration stability reduces to Gaussian blocking. On the Goepfert–Mack fixed-Debye-mass trajectory $\lambda_3 a \simeq c_0/(2\log(1/a))$, $c_0 \approx 4.99$, the monopole density per physical volume diverges. In four dimensions $t = g^2$ does not decrease, so the analogous bound grows like $(L/a)^4$ and is useless. Proving that the route itself fails would need an unproved dilute-gas lower bound.

Full note: <https://arivero.github.io/navs tokgap/notes/villain-monopole-refinement.html>

One 4D lattice cut gives six regular one-loop coupling shifts; $\log 2$ needs matched determinants

Four-dimensional parallel insertions: one-loop shifts and the logarithm

Note four-dimensional-parallel-log. Interest 6/10, correctness 8/10; conditional calculation.

For pure $SU(N)$ lattice gauge theory in four dimensions with heat-kernel (Villain-like) plaquette weights and heat times $t\tau_{\mu\nu}$, the note cuts one direction at fraction s and integrates the new mid-links. In the formal bulk small-

coupling expansion, the three cut-plane coupling shifts are

$$\delta_t^{1j} = \frac{C_A \sigma t \tau_{1j}}{12} \int [1 - M_j G_{jj}(k)]$$

with $\sigma = s(1 - s)$, and the three transverse-plane shifts are $C_A t \tau_{jk} (-\frac{1}{24} + \mathcal{I}_{jk})$, where $\mathcal{I}_{jk}$ is an explicit magnetic-Weyl resolvent integral over 3×3 matrices. The conditional mid-space propagator has a positive bridge mass, so every one-step integral is regular at zero momentum and carries no infrared logarithm. A finite-torus winding term is bounded by $7V_3(3/7)^{n_*}/n_*$. The universal running $u(2a) - u(a) = -\frac{11N}{24\pi^2} \log 2$ is recovered only conditionally (Theorem 2), when the full generated action is retained, background Ward identities and continuum determinant matching hold, and endpoint matching coefficients agree or stay bounded. Four applications of pristine one-step coefficients do not establish it. The note proves no remainder estimates, no continuum measure and no mass gap.

Full note: <https://arivero.github.io/navs tokgap/notes/four-dimensional-parallel-log.html>

Continuous-time polymer expansion gives $SU(3)$ Kogut–Susskind gap $\geq \frac{4}{3}g^2\hbar c/a$ for $g^2 \geq 388$

Continuous-time expansion for Kogut–Susskind: $SU(3)$ gapped for $g^2 \geq 388$, gap approaching $(8/3)g^2\hbar c/a$

Note kogut-susskind-strong-coupling-explicit. Interest 6/10, correctness 8/10; proved theorem.

For the $SU(3)$ Kogut–Susskind lattice Hamiltonian in units $\hbar c/a$, the note expands e^{-tH} by Duhamel in the plaquette term, keeping the electric evolution exact between insertions. The electric term is diagonal in the character basis with each excited link costing at least $\varepsilon = (g^2/2)C_2(f) = 2g^2/3$. Gauss’s law forces every nonempty intermediate excited set to contain at least four links. Histories form a continuous-time polymer gas, and the measure-space Kotecký–Preiss criterion with weight $a(\gamma) = n + \lambda|\text{supp } \gamma|$, $\lambda = \varepsilon(1 - \theta)$, converges when the per-insertion factor is small. The entropy of choosing the next plaquette cancels against the energy denominator link by link. Counting all connected histories by a spanning tree gives a rigorous volume-uniform gap $\Delta_{\text{KS}} \geq \frac{8}{3}g^2(1 - \theta)\hbar c/a$ for $g^4 \geq 75168/\theta$, for example $\frac{4}{3}g^2$ at $g^2 \geq 388$; a restricted adjacent-growth class gives $g^2 \geq 79$. As $g \rightarrow \infty$ the bound approaches the four-link flux-loop energy. This is a strong-coupling

lattice statement only, with no weak-coupling or continuum content. Constants are explicit throughout, in units of $\hbar c/a$.

Full note: <https://arivero.github.io/navstokgap/notes/kogut-susskind-strong-coupling-explicit.html>

Feynman–Bijl bounds the lattice gauge gap; $U(1)$ gap is bounded by a plaquette-sine structure factor

Upper bounds on the lattice gap: the Feynman–Bijl inequality, the abelian structure factor and the non-abelian dressing obstruction

Note lattice-gap-upper-bounds. Interest 6/10, correctness 8/10; mixed theorem.

For the Kogut–Susskind lattice Hamiltonian, the single-mode (Feynman–Bijl) inequality gives, for every real gauge-invariant link function f with zero mean,

$$\Delta^{\text{phys}} \leq \frac{\hbar c g^2}{2a} \sum_{\ell} \langle |\nabla_{\ell} f|^2 \rangle / \langle f^2 \rangle$$

. For a plaquette this is

$$\Delta \leq \hbar c g^2 N / (a \text{Var Re tr } U_p)$$

, and at strong coupling, with Haar variance $\frac{1}{2}$ for $SU(N \geq 3)$, the gap is $g^2 \hbar c/a$ times a number bounded above by $4N$ and below by the strong-coupling theorem. For compact $U(1)$, the gauge-invariant operator

$\sum \varphi_\ell E_\ell$ and an uncertainty relation give $\Delta \leq 2\hbar c \hat{S}(k)/(ag^2 \langle \cos \theta_p \rangle)$, where $\hat{S}$ is the plaquette-sine structure factor, so a Coulomb phase with $\hat{S}(k) \rightarrow 0$ is gapless. For a semisimple group, no nonzero operator linear in the electric field with c-number coefficients commutes with Gauss's law, so the abelian trial excitation needs a Wilson-line dressing whose electric cost does not vanish at long wavelength. These are upper bounds only; plaquette bounds diverge in the continuum, and no lower bound is proved. Explicit constants are kept throughout.

Full note: <https://arivero.github.io/navs/tokgap/notes/lattice-gap-upper-bounds.html>

Gaps are unit multiples of fixed constants; Yang–Mills matrix-model gap

$\propto \hbar^{4/3} g^{2/3} m^{-2/3}$ **needs both commutators**

A gap is a scale the classical theory lacks: solved low-dimensional mass gaps and the positive-action question

Note low-dimensional-mass-gap. Interest 6/10, correctness 8/10; mixed theorem.

The note compares the Yang–Mills mass gap with the programme's positive action floor. A dimensional theorem shows that the infimum of an observable fixed by homogeneous relations is a monomial in the theory's fixed con-

stants times a function of dimensionless ratios, and is 0 or ∞ when no such monomial exists. Classical Yang–Mills and Newton’s constant-force comparison both lack the needed unit, and a similarity realizes zero. For homogeneous potentials of degree k , spectra scale as $\bar{h}^{2k/(k+2)} \lambda^{2/(k+2)} m^{-k/(k+2)}$. Following Simon, the $SU(2)$ Yang–Mills matrix model has compact resolvent and a simple ground state with gap $\delta_1 \bar{h}^{4/3} g^{2/3} m^{-2/3}$, which vanishes as either $\bar{h}$ or g goes to zero; on a torus of side L the zero-mode gap is $\delta_1 g^{2/3} \bar{h} c/L$, positive for every L but not uniform. Free and abelian fields generate no scale, two-dimensional Yang–Mills gives flux energies growing with L , and the inverse-square potential shows a scale forced by self-adjointness. The note distinguishes supplied gaps from generated ones that need two non-commuting scale-free structures. It labels as conjecture the hope that a consistency requirement forces Newton’s action floor.

Full note: <https://arivero.github.io/navs tokgap/notes/low-dimensional-mass-gap.html>

Formal one-loop halving: cut couplings rise, transverse coupling falls; flat-holonomy winding term obstructs fixed-size $P(\alpha)$

$SU(2)$ on the full mid-plane: the relative-order- t halving defect

Note su2-midplane-order-t. Interest 6/10, correctness 8/10; conditional calculation.

The note studies one halving of direction 1 in three-dimensional $SU(2)$ lattice gauge theory with heat-kernel weights at coarse heat time $t = \lambda_3 a$. The new mid-plane links are integrated out. A formal one-loop Laplace expansion of the normalized halving defect gives coupling shifts $\delta_t^{12} = \delta_t^{13} = \frac{t}{24} \int q/R + O(t^2) > 0$ for the two planes containing the halved direction. The transverse plane gets $\delta_t^{23} = t(-\frac{1}{12} + 2\mathcal{I}) + O(t^2) < 0$, where $\mathcal{I}$ is an explicit Brillouin-zone integral proved negative from the sign of its numerator, without numerical evaluation. Here $q = 2 - 2 \cos k_2$, $r = 2 - 2 \cos k_3$ and $R = 8 + q + r$. Gauge invariance and reflection symmetry leave only dimension-six and higher bulk operators beyond these. On a finite periodic plane with flat commuting holonomies, an order-one winding determinant $\Omega_N(\theta)$ appears. It falsifies the flux-only perturbative hypothesis $P(\alpha)$ at fixed plane size and is $O(t^{-1}e^{-c/t})$ when the plane has at least c_0/t sites per side. For $SU(N)$ every bulk coefficient is multiplied by $N/2$. The note does not prove the volume-uniform normalized estimate needed for a theorem. It states that estimate precisely, and it does not address iteration or the infrared gap.

Full note: <https://arivero.github.io/navs tokgap/notes/su2-midplane-order-t.html>

Exact bridge character formula at any cut; images are central iff $(1 - s)H$ is a coweight

The bridge midpoint on a compact group at any cut: windings and the centre

Note sun-midpoint-centre. Interest 6/10, correctness 8/10; proved theorem.

Take a compact connected simply connected Lie group with bi-invariant metric, and the heat-kernel bridge of time t from e to e^{iX} , cut at fraction s . The expected character of the cut point in any irreducible representation is an exact finite sum over weights. Each weight term carries an image sum over the coroot lattice with Weyl-polynomial amplitudes $\pi(iZ/t - (1 - s)\kappa)$ and Gaussian weights $e^{-|X - 2\pi H|^2/2t}$. A several-cut version holds with the Brownian-bridge covariance. The formula reduces to the $U(1)$ winding mixture and to the earlier exact $SU(2)$ midpoint results. An image's phase is the same for all weights exactly when $(1 - s)H$ lies in the coweight lattice, and it is then a central character. At $s = p/q$ these central images reach precisely the q -torsion of the centre, $\mathbb{Z}_{\gcd(q, N)}$ for $SU(N)$. Halvings therefore never produce $SU(3)$ triality phases, while trisections give ω, ω^2 at relative weight $e^{-24\pi^2/t}$. Images are signed amplitudes, not probabilities of winding or vortex sectors. The note proves no per-cell bound uniform in boundary data and no multi-step statement.

Full note: <https://arivero.github.io/navs tokgap/notes/sun-midpoint-centre.html>